PROJECTIVE INVARIANCE AND ONE-LOOP EFFECTIVE ACTION IN AFFINE-METRIC GRAVITY INTERACTING WITH SCALAR FIELD

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Abstract

We investigate the influence of the projective invariance on the renormalization properties of the theory. One-loop counterterms are calculated in the most general case of interaction of gravity with scalar field.

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1 Introduction

The construction of quantum theory of gravity is an unresolved problem of modern theoretical physics. It is well know that the Einstein theory of gravity is not renormalizable in an ordinary sense [1] – [3]. Therefore, one needs to modify the theory or to show, that the difficulties presently encountered in the theory are only artifacts of perturbation theory. The simplest method of modifying the Einstein theory is to introduce terms quadratic in the curvature tensor in the action of the theory.

$$L_{gr} = \left(-\frac{1}{k^2}R + \alpha R^2_{\mu\nu} + \beta R^2\right)\sqrt{-g} \tag{1}$$

This theory is renormalizable and asymptotically free but it is not unitary because the ghosts and tachyons are present in the spectrum of the theory [4] – [6]. It should be noted that it is impossible to restore the unitarity of the theory by means of loop corrections or adding an interaction with matter fields [7] , [8]. Hence, one needs to use a new method in order to construct the theory of gravity.

Among various methods of constructing the quantum theory of gravity one should emphasize the gauge approach as the most promising [9] – [12]. In gauge treatment of gravity there are two sets of dynamical variables, namely, the vierbein $h^a{}_\mu(x)$ and local Lorentz connection $\omega^a{}_{b\mu}(x)$ or metric $g_{\mu\nu}(x)$ and affine connection $\Gamma^\sigma{}_{\mu\nu}(x)$. The theory based on the first set of variables is called the Poincarè gauge gravitational theory with the structure group P_{10} [13], [14]. A curvature tensor $R^a{}_{b\mu\nu}(\omega)$ and a torsion tensor $Q^a{}_{\mu\nu}(h,\omega)$, which are the strength tensors of the Poincarè gauge gravitational theory, are defined by the following relations:

$$\begin{aligned} R^a{}_{b\mu\nu}(\omega) &= \partial_\mu \omega^a{}_{b\nu} - \partial_\nu \omega^a_{b\mu} + \omega^a{}_{c\mu}\omega^c_{b\nu} - \omega^a{}_{c\nu}\omega^c_{b\mu} && (2)\\ Q^a{}_{\mu\nu}(h,\omega) &= -\frac{1}{2}\left(\partial_\mu h^a{}_\nu - \partial_\nu h^a{}_\mu + \omega^a{}_{c\nu}h^c{}_\mu - \omega^a{}_{c\mu}h^c{}_\nu\right) && (3)\end{aligned}$$

The theory based on the second set of variables is called the affine gauge gravitational theory with the structure gauge group $GA(4,R)$ [15] – [17]. The strength tensor of the theory is the curvature tensor $\tilde{R}^\sigma{}_{\lambda\mu\nu}(\Gamma)$ defined as:

$$\tilde{R}^\sigma{}_{\lambda\mu\nu}(\Gamma) = \partial_\mu \Gamma^\sigma{}_{\lambda\nu} - \partial_\nu \Gamma^\sigma_{\lambda\mu} + \Gamma^\sigma{}_{\alpha\mu}\Gamma^\alpha{}_{\lambda\nu} - \Gamma^\sigma{}_{\alpha\nu}\Gamma^\alpha_{\lambda\mu} \tag{4}$$

The Lagrangian of a gauge theory is built out of terms quadratic in the strength tensor of fields. In the Poincarè or affine gauge theories the Lagrangians are defined as :

$$\begin{aligned} L_{P_{10}} &= \left(\frac{A_i}{k^2}Q^2(h,\omega) + B_j R^2(\omega)\right)\sqrt{-g} && (5)\\ L_{GA(4,R)} &= C_j \tilde{R}^2(\Gamma)\sqrt{-g} && (6)\\ & && (7)\end{aligned}$$

where A_i, B_j, C_j are arbitrary constants and R^2, Q^2 are now a symbolic notation for the contractions of the curvature tensors or the torsion tensors respectively.

At the present time there are a lot of papers concerning the classical problems of these theories [18] – [23]. For example, it is possible to find some coefficients A_i and B_j in the Poincarè gauge gravitational theory in order to obtain a unitary model [24] – [26]. However, the renormalizability properties of the theories have been studied insufficiently [27] – [29].

In the affine-metric theory of gravity there are models possessing an extra projective symmetry. By the projective invariance we mean that the action is invariant under the following transformation of fields:

$$\begin{array}{rcl} x^\mu & \rightarrow & {}'x^\mu = x^\mu \\ g_{\mu\nu}(x) & \rightarrow & {}'g_{\mu\nu}(x) = g_{\mu\nu}(x) \\ \Phi_{mat}(x) & \rightarrow & {}'\Phi_{mat}(x) = \Phi_{mat}(x) \\ \Gamma^\sigma{}_{\mu\nu}(x) & \rightarrow & {}'\Gamma^\sigma{}_{\mu\nu}(x) = \Gamma^\sigma{}_{\mu\nu}(x) + \delta^\sigma_\mu C_\nu(x) \end{array} \tag{8}$$

where $C_\nu(x)$ is an arbitrary vector.

The classical properties of models with the projective invariance have been discussed in papers [30], [31]. However, the quantum properties of the projective invariance have not been investigated. It should be noted, that the presence of an additional symmetry in the theory may improve the renormalization properties of the theory. For example, because of the presence of supersymmetry, the terms violating the renormalizability of supergravity, show up only in higher loops. So, the projective invariance may have the considerable role for the renormalizability of the theory. In order to investigate the influence of the projective invariance on renormalizability of the theory one needs to calculate the counterterms in some model possessing the projective invariance. The simplest model of this type is the model with the Lagrangian:

$$L_{gr} = -\frac{1}{k^2}\tilde{R}(\Gamma)\sqrt{-g} \tag{9}$$

But because of the degeneracy of the four-dimensional space-time [32], the terms violating the renormalizability of the theory arise only at two-loop level. The two-loop calculations are very cumbersome. Since we would like to restrict ourselves to the one-loop calculations and to investigate the influence of the projective invariance on the renormalizability of the models, we consider the interaction of the gravity with a matter field.

$$L_{gr} = \left(\left(\xi\varphi^2 - \frac{1}{k^2}\right)\tilde{R}(\Gamma) + \frac{2}{k^2}\Lambda + \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\right)\sqrt{-g} \tag{10}$$

where Λ is a cosmological constant.

We consider $\Gamma^{\sigma}{}_{\mu\nu}(x), \phi(x), g_{\mu\nu}(x)$ as independent dynamical fields. This model is invariant under the projective transformation (8) and general coordinate transformation:

$$
\begin{aligned}
x^{\mu} &\rightarrow {}'x^{\mu} = x^{\mu} + \xi^{\mu}(x) \\
g_{\mu\nu}(x) &\rightarrow {}'g_{\mu\nu}(x) = g_{\mu\nu}(x) - \partial_{\mu}\xi^{\alpha}g_{\alpha\nu}(x) - \partial_{\nu}\xi^{\alpha}g_{\alpha\mu}(x) - \xi^{\alpha}\partial_{\alpha}g_{\mu\nu}(x) \\
\varphi(x) &\rightarrow {}'\varphi(x) = \varphi(x) - \xi^{\alpha}\partial_{\alpha}\varphi(x) \\
\Gamma^{\sigma}{}_{\mu\nu}(x) &\rightarrow {}'\Gamma^{\sigma}{}_{\mu\nu}(x) = \Gamma^{\sigma}{}_{\mu\nu}(x) - \partial_{\mu}\xi^{\alpha}\Gamma^{\sigma}{}_{\alpha\nu}(x) - \partial_{\nu}\xi^{\alpha}\Gamma^{\sigma}{}_{\mu\alpha}(x) \\
&\qquad + \partial_{\alpha}\xi^{\sigma}\Gamma^{\alpha}{}_{\mu\nu}(x) - \xi^{\alpha}\partial_{\alpha}\Gamma^{\sigma}{}_{\mu\nu}(x) - \partial_{\mu\nu}\xi^{\sigma} \qquad (11)
\end{aligned}
$$

The main aim of our paper is to research the influence of the projective invariance on the renormalization properties of the theory. In particular, we consider the following problems in the next section:

1. A necessity of introducing the term fixing the projective invariance at the quantum level.
2. The presence of the ghosts connected with the projective invariance.
3. The addition of the "projective" ghosts to the one-loop effective action

We use the following notations:

$$c = \hbar = 1; \qquad \mu, \nu = 0, 1, 2, 3; \qquad k^2 = 16\pi G$$

$$\tilde{R}_{\mu\nu}(\Gamma) = \tilde{R}^{\sigma}{}_{\mu\sigma\nu}(\Gamma), \qquad \tilde{R}(\Gamma) = \tilde{R}{}_{\mu\nu}(\Gamma)g^{\mu\nu}, \qquad (-g) = det(g_{\mu\nu})$$

The objects marked by the tilde $\tilde{}$ are constructed by means of the affine connection $\Gamma^{\sigma}{}_{\mu\nu}$. The others are the Riemannian objects.

2 One-loop counterterms

For calculating the one-loop effective action we use the background field method [33], [34]. In accordance with this method all dynamical variables are rewritten as the sum of the classical and quantum parts. In general case, the dynamical variables in the affine-metric theory are $\Gamma^{\sigma}_{\mu\nu}, \bar{g}_{\mu\nu} = g_{\mu\nu}(-g)^{r}, \bar{\varphi} = \varphi(-g)^{s}$, where r, s are arbitrary numbers satisfying the only condition: $r \neq -\frac{1}{4}$. The one-loop counterterms on the mass-shell do not depend on the value of r and s. To simplify our calculation we use the following values: $r = s = 0$.

The fields $\Gamma^{\sigma}{}_{\mu\nu}, g_{\mu\nu}, \varphi$ are now rewritten according to

$$
\begin{aligned}
\Gamma^{\sigma}{}_{\mu\nu} &= \Gamma^{\sigma}_{\mu\nu} + k\gamma^{\sigma}{}_{\mu\nu} \\
g_{\mu\nu} &= g_{\mu\nu} + kh_{\mu\nu} \\
\varphi &= \frac{1}{k}\varphi + \phi \qquad (12)
\end{aligned}
$$

where $\Gamma^{\sigma}{}_{\mu\nu}, g_{\mu\nu}, \varphi$ are the classical parts satisfying the following equations

$$
\begin{aligned}
\frac{\delta S}{\delta \Gamma^{\sigma}{}_{\mu\nu}} = 0 \ \ &\Rightarrow \ \ D^{\sigma}{}_{\mu\nu} = -\frac{1}{2}\frac{1}{\alpha(\varphi)}\partial_\lambda \alpha(\varphi)(g^{\lambda\sigma}g_{\mu\nu} - \delta^{\lambda}_{\mu}\delta^{\sigma}_{\nu}) + \delta^{\sigma}_{\mu}C_\nu \\
\frac{\delta S}{\delta g_{\mu\nu}} = 0 \ \ &\Rightarrow \ \ -\alpha(\varphi)\tilde{R}_{(\mu\nu)}(\Gamma) = \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi + \Lambda g_{\mu\nu} \\
\frac{\delta S}{\delta \varphi} = 0 \ \ &\Rightarrow \ \ 2\xi\varphi\tilde{R}(\Gamma) - g^{\mu\nu}\nabla_\mu\nabla_\nu\varphi = 0 \qquad (13)
\end{aligned}
$$

where

$$
\begin{aligned}
D^{\sigma}{}_{\mu\nu} &= \Gamma^{\sigma}{}_{\mu\nu} - g^{\sigma\lambda}\frac{1}{2}\left(-\partial_\lambda g_{\mu\nu} + \partial_\mu g_{\nu\lambda} + \partial_\nu g_{\mu\lambda}\right) \\
\alpha(\varphi) &= \xi\varphi^2 - 1
\end{aligned}
$$

C_ν is an arbitrary vector.

The action (10) expanded as a power series in the quantum fields (12) defines the effective action for calculating the loop counterterms. The one-loop effective Lagrangian quadratic in the quantum fields is:

$$
\begin{aligned}
L_{eff} \ = \ & \Bigg(\alpha(\varphi)\frac{1}{2}\gamma^{\sigma}{}_{\mu\nu}F_{\sigma}{}^{\mu\nu}{}_{\lambda}{}^{\alpha\beta}\gamma^{\lambda}{}_{\alpha\beta} + \alpha(\varphi)\frac{1}{2}h_{\mu\nu}h_{\sigma\lambda}D^{\mu\nu\sigma\lambda} - \frac{1}{4}h_{\alpha\beta}h_{\mu\nu}\Lambda P^{-1\alpha\beta\mu\nu} \\
+ \ & \xi\phi^2 R + \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}\alpha(\varphi)h_{\alpha\beta}P^{-1\alpha\beta\mu\nu}\Big(B_{\lambda}{}^{\epsilon\tau\sigma}{}_{\mu\nu}\nabla_\sigma + \triangle_{\lambda}{}^{\epsilon\tau}{}_{\mu\nu}\Big)\gamma^{\lambda}{}_{\epsilon\tau} \\
+ \ & 2\xi\varphi\phi\bigg(\Big(B_{\lambda}{}^{\epsilon\tau\sigma}\nabla_\sigma + \triangle_{\lambda}{}^{\epsilon\tau}\Big)\gamma^{\lambda}{}_{\epsilon\tau} - \frac{1}{2}h_{\alpha\beta}P^{-1\alpha\beta\mu\nu}R_{\mu\nu}\bigg) \\
+ \ & \frac{1}{2}\nabla_\mu\varphi\nabla_\nu\varphi\Big(h^{\mu\lambda}h^{\nu}{}_{\lambda} - \frac{1}{2}hh^{\mu\nu} - \frac{1}{8}h_{\alpha\beta}h_{\sigma\lambda}g^{\mu\nu}P^{-1\alpha\beta\sigma\lambda}\Big) \\
- \ & \frac{1}{2}\nabla_\mu\varphi\nabla_\nu\phi h_{\alpha\beta}P^{-1\alpha\beta\mu\nu}\Bigg)\sqrt{-g} \qquad (14)
\end{aligned}
$$

where

$$
\begin{aligned}
P^{-1\alpha\beta\mu\nu} &= g^{\alpha\mu}g^{\beta\nu} + g^{\alpha\nu}g^{\beta\mu} - g^{\alpha\beta}g^{\mu\nu} \\
\triangle_{\lambda}{}^{\alpha\beta}{}_{\mu\nu} &\equiv D^{\alpha}{}_{\mu\nu}\delta^{\beta}_{\lambda} + D_\lambda\delta^{\alpha}_{\mu}\delta^{\beta}_{\nu} - D^{\alpha}{}_{\mu\lambda}\sigma^{\beta}_{\nu} - D^{\beta}{}_{\lambda\nu}\delta^{\alpha}_{\mu} \\
B_{\lambda}{}^{\alpha\beta\sigma}{}_{\mu\nu} &= \delta^{\sigma}_{\lambda}\delta^{\alpha}_{\mu}\delta^{\beta}_{\nu} - \delta^{\beta}_{\lambda}\delta^{\alpha}_{\mu}\delta^{\sigma}_{\nu} \\
F_{\alpha}{}^{\beta\lambda}{}_{\mu}{}^{\nu\sigma} &= g^{\beta\lambda}\delta^{\nu}_{\alpha}\delta^{\sigma}_{\mu} - g^{\beta\sigma}\delta^{\nu}_{\alpha}\delta^{\lambda}_{\mu} + g^{\nu\sigma}\delta^{\lambda}_{\alpha}\delta^{\beta}_{\mu} - g^{\lambda\nu}\delta^{\sigma}_{\alpha}\delta^{\beta}_{\mu} \\
\triangle_{\lambda}{}^{\epsilon\tau} &= \triangle_{\lambda}{}^{\epsilon\tau}{}_{\mu\nu}g^{\mu\nu} \\
B_{\lambda}{}^{\epsilon\tau\sigma} &= B_{\lambda}{}^{\alpha\beta\sigma}{}_{\mu\nu}g^{\mu\nu} \\
D^{\alpha\beta\mu\nu} &= 2R^{\alpha\mu}g^{\beta\nu} - RP^{\alpha\beta\mu\nu} - R^{\alpha\beta}g^{\mu\nu} \qquad (15)
\end{aligned}
$$

Now we may define the propagators of the quantum fields $\gamma^{\sigma}{}_{\mu\nu}, h_{\mu\nu}, \phi$. The propagator of the quantum field $\gamma^{\sigma}{}_{\mu\nu}$ satisfies two conditions:

$$F^{-1\sigma}{}_{\mu\nu}{}^{\lambda}{}_{\alpha\beta} = F^{-1\lambda}{}_{\alpha\beta}{}^{\sigma}{}_{\mu\nu} \tag{16}$$

$$F^{-1\sigma}{}_{\mu\nu}{}^{\lambda}{}_{\alpha\beta} F_{\lambda}{}^{\alpha\beta}{}_{\rho}{}^{\tau\epsilon} = \delta^{\sigma}_{\rho}\delta^{\tau}_{\mu}\delta^{\epsilon}_{\nu} \tag{17}$$

However, because of the projective invariance of the effective Lagrangian (14) the propagator does not exist. Under transformation (8) the quantum part of the connection transforms as

$$\gamma^{\sigma}{}_{\mu\nu}(x) \to' \gamma^{\sigma}{}_{\mu\nu}(x) = \gamma^{\sigma}{}_{\mu\nu}(x) + \delta^{\sigma}_{\mu}C_{\nu}(x) \tag{18}$$

In order to fix the projective invariance we use the following condition:

$$f_{\lambda} = \Big(B_1 g_{\lambda\sigma} g^{\alpha\beta} + B_2 \delta^{\alpha}_{\sigma}\delta^{\beta}_{\lambda} + B_3 \delta^{\beta}_{\sigma}\delta^{\alpha}_{\lambda}\Big)\gamma^{\sigma}{}_{\alpha\beta} \equiv f_{\lambda\sigma}{}^{\alpha\beta}\gamma^{\sigma}{}_{\alpha\beta} \tag{19}$$

$$L_{gf} = \frac{1}{2} f_{\mu} f^{\mu} \tag{20}$$

where B_j are the constants satisfying the only condition:

$$B_1 + B_3 + 4B_2 \neq 0 \tag{21}$$

The action of the projective ghosts defined by the standard way has the following structure:

$$L_{gh} = \overline{\chi}^{\mu} g_{\mu\nu}(-g)^{\alpha}\chi^{\nu} \tag{22}$$

where

$\overline{\chi}^{\mu}, \chi^{\nu}$ are the grassmann variables; α is a constant.

The one-loop contribution of the projective ghosts to the effective action is proportional to the $\delta^4(0)$. In the dimensional regularization $[\delta^4(0)]_R = 0$ and the contribution of the projective ghosts to the one-loop counterterms is equal to zero.

Now, we must change the equation (17). The propagator of the quantum field $\gamma^{\sigma}{}_{\mu\nu}$ satisfies equation (16) and new condition:

$$F^{-1\sigma}{}_{\mu\nu}{}^{\lambda}{}_{\alpha\beta} \overline{F}_{\lambda}{}^{\alpha\beta}{}_{\rho}{}^{\tau\epsilon} = \delta^{\sigma}_{\rho}\delta^{\tau}_{\mu}\delta^{\epsilon}_{\nu} \tag{23}$$

where

$$\begin{aligned}
\overline{F}_{\sigma}{}^{\alpha\beta}{}_{\lambda}{}^{\mu\nu} &= F_{\sigma}{}^{\alpha\beta}{}_{\lambda}{}^{\mu\nu} + f_{\tau\sigma}{}^{\alpha\beta} f^{\tau}{}_{\lambda}{}^{\mu\nu} \\
&= g^{\mu\nu}\delta^{\alpha}_{\lambda}\delta^{\beta}_{\sigma}(1 + B_1 B_3) + g^{\alpha\beta}\delta^{\mu}_{\sigma}\delta^{\nu}_{\lambda}(1 + B_1 B_3) - g^{\nu\alpha}\delta^{\mu}_{\sigma}\delta^{\beta}_{\lambda} - g^{\mu\beta}\delta^{\nu}_{\sigma}\delta^{\alpha}_{\lambda} \\
&+ B_1 B_2 g^{\alpha\beta}\delta^{\nu}_{\sigma}\delta^{\mu}_{\lambda} + B_1 B_2 g^{\mu\nu}\delta^{\alpha}_{\sigma}\delta^{\beta}_{\lambda} + B_1^2\ g_{\sigma\lambda} g_{\mu\nu} g_{\alpha\beta} + B_3^2\ g^{\alpha\mu}\delta^{\nu}_{\lambda}\delta^{\beta}_{\sigma} + \\
&+ B_2 B_3 g^{\mu\beta}\delta^{\alpha}_{\sigma}\delta^{\nu}_{\lambda} + B_2 B_3 g^{\alpha\nu}\delta^{\beta}_{\sigma}\delta^{\mu}_{\lambda} + B_2^2\ g^{\nu\beta}\delta^{\alpha}_{\sigma}\delta^{\mu}_{\lambda}
\end{aligned} \tag{24}$$

Having solved equations (16),(23) we obtain the following result:

$$
\begin{aligned}
F^{-1\alpha\ \ \mu}_{\ \ \ \beta\sigma\ \nu\lambda} &= -\frac{1}{4}g^{\alpha\mu}g_{\beta\sigma}g_{\nu\lambda}+\frac{1}{2}g^{\alpha\mu}g_{\beta\nu}g_{\sigma\lambda}-\frac{1}{4}g_{\nu\beta}\delta^{\mu}_{\lambda}\delta^{\alpha}_{\sigma}\\
&+ \frac{1}{4}\Big(g_{\nu\lambda}\delta^{\mu}_{\beta}\delta^{\alpha}_{\sigma}+g_{\beta\sigma}\delta^{\alpha}_{\nu}\delta^{\mu}_{\lambda}\Big)-\frac{1}{2}\Big(g_{\nu\sigma}\delta^{\alpha}_{\lambda}\delta^{\mu}_{\beta}+g_{\beta\lambda}\delta^{\mu}_{\sigma}\delta^{\alpha}_{\nu}\Big)\\
&+ \frac{1}{4}\left(\frac{B_1-B_3+2B_2}{B_1+B_3+4B_2}\right)\Big(g_{\nu\lambda}\delta^{\mu}_{\sigma}\delta^{\alpha}_{\beta}+g_{\beta\sigma}\delta^{\alpha}_{\lambda}\delta^{\mu}_{\nu}\Big)\\
&+ \frac{1}{4}\left(\frac{B_3-B_1+2B_2}{B_1+B_3+4B_2}\right)\Big(g_{\beta\lambda}\delta^{\mu}_{\nu}\delta^{\alpha}_{\sigma}+g_{\beta\nu}\delta^{\alpha}_{\beta}\delta^{\mu}_{\lambda}\Big)\\
&+ \frac{1}{4}\frac{1}{(B_1+B_3+4B_2)^2}(4-B_1^2-B_3^2-12B_2^2+\\
&+ 10B_1B_3-4B_1B_2-4B_2B_3)g_{\sigma\lambda}\delta^{\mu}_{\nu}\delta^{\alpha}_{\beta} \qquad (25)
\end{aligned}
$$

To get the diagonal form of the effective Lagrangian we are to replace the dynamical variables in the following way:

$$
\begin{aligned}
\gamma^{\sigma}_{\ \mu\nu}\to\tilde{\gamma}^{\sigma}_{\ \mu\nu}=\gamma^{\sigma}_{\ \mu\nu}+\frac{1}{2}F^{-1\sigma\ \ \ \lambda}_{\ \ \ \mu\nu\ \alpha\beta}\Big(B_{\lambda\ \ \ \rho\epsilon}^{\ \alpha\beta\tau}\nabla_{\tau}-\triangle_{\lambda\ \ \rho\epsilon}^{\ \alpha\beta}\Big)P^{-1\rho\epsilon\kappa\upsilon}h_{\kappa\upsilon}&\\
+\frac{1}{2}F^{-1\sigma\ \ \ \lambda}_{\ \ \ \mu\nu\ \alpha\beta}P^{-1\rho\epsilon\kappa\upsilon}h_{\kappa\upsilon}B_{\lambda\ \ \ \rho\epsilon}^{\ \alpha\beta\eta}\frac{1}{\alpha(\varphi)}\nabla_{\eta}\alpha(\varphi)&\\
+2\xi\phi\varphi\frac{1}{\alpha(\varphi)}F^{-1\sigma\ \ \ \lambda}_{\ \ \ \mu\nu\ \alpha\beta}\triangle_{\lambda}^{\ \alpha\beta}-\frac{2\xi}{\alpha(\varphi)}F^{-1\sigma\ \ \ \lambda}_{\ \ \ \mu\nu\ \alpha\beta}B_{\lambda}^{\ \alpha\beta\tau}\nabla_{\tau}(\phi\varphi)& \qquad (26)
\end{aligned}
$$

This replacement does not change the functional measure:

$$
det\frac{\partial\tilde{\gamma}}{\partial\gamma}=1 \qquad (27)
$$

We don't give the details of the cumbersome one-loop calculations that have been performed by means of the special REDUCE package program created by K.V.Stepanyantz. One should note, that we violate the invariance of the action (14) under the general coordinate transformation by means of the following gauge [35]:

$$
F_{\mu}=\nabla_{\nu}h_{\mu}^{\ \nu}-\frac{1}{2}\nabla_{\mu}h-\frac{2\xi\varphi}{\alpha(\varphi)}\nabla_{\mu}\phi \qquad (28)
$$

$$
L_{gf}=\frac{1}{2}F_{\mu}F^{\mu} \qquad (29)
$$

The action of the coordinate ghost is

$$
L_{gh}=\overline{c}^{\mu}\Big(g_{\mu\nu}\nabla^2+R_{\mu\nu}-\frac{2\xi\varphi}{\alpha(\varphi)}(\nabla_{\nu}\varphi)\nabla_{\mu}-\frac{2\xi\varphi}{\alpha(\varphi)}(\nabla_{\mu}\nabla_{\nu}\varphi)\Big)c^{\nu} \qquad (30)
$$

The one-loop counterterms on the mass-shell including the contributions of the quantum and ghost fields are

$$\triangle\Gamma^1_\infty = -\frac{1}{32\pi^2\varepsilon}\int d^4x\sqrt{-g}\Big(\frac{71}{60}\Big(R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}-4R_{\mu\nu}R^{\mu\nu}+R^2\Big)$$
$$+\frac{203}{40}R^2+\frac{\Lambda^2}{\alpha^2(\varphi)}\Big(\frac{463}{5}+52\xi^2\Big)$$
$$+\Lambda R\Big(\frac{1}{\alpha(\varphi)}\Big(5\xi^2-\frac{4}{3}\xi+\frac{463}{10}\Big)+\frac{\xi^2\varphi^2}{\alpha^2(\varphi)}\Big(75\xi+\frac{20}{3}\Big)-\frac{700}{3}\frac{\xi^4\varphi^4}{\alpha^3(\varphi)}\Big)\Big) \tag{31}$$

3 Conclusion

In our paper we have investigated the influence of the projective invariance on the renormalizability of the theory. It turns out that:

1. In order to define the propagator of the quantum fields $\gamma^\sigma_{\mu\nu}$ one needs to fix the projective invariance.

2. The gauge fixing term (19) has the algebraic structure, that is it does not contain derivatives of the fields.

3. The action of the projective ghosts (22) has also the algebraic structure. The one-loop contribution of the projective ghosts is proportional to the $\delta^4(0)$ Hence its contribution is equal to zero in the dimensional regularization.

4. The theory involved is not renormalizable. The term violating the renormalizability of the theory is equal to the R^2. It is easy to show [1], that the expression $\int d^4x\sqrt{-g}\Big(R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}-4R_{\mu\nu}R^{\mu\nu}+R^2\Big)$ is equal to the $\int d^4x\partial_\mu j^\mu$. Hence, we can neglect the contribution of this term to the one-loop counterterms in space-time without boundaries.

5. The renormalizability of the theory is not affected by the presence of the projective invariance.

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